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Fermi-Dirac statistics plus liquid description of quark partons

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Abstract

A previous approach with Fermi-Dirac distributions for fermion partons is here improved to comply with the expected low x behaviour of structure functions. We are so able to get a fair description of the unpolarized and polarized structure functions of the nucleons as well as of neutrino data. We cannot reach definite conclusions, but confirm our suspicion of a relationship between the defects in Gottfried and spin sum rules.

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1 Introduction

The experimental data on the unpolarized and polarized structure functions of the nucleons suggest a role of Pauli principle in relating the shapes and the first momenta of the distributions of the various quark parton species, for which Fermi-Dirac distributions in the x variable have been proposed. Here, to comply with the expected behaviour for the structure functions in the limit $x \to 0$, we add a *liquid* unpolarized component dominating the very low x region and not contributing to quark parton model sum rules (QPMSR). Section 2 deals with the motivations for our description of parton distributions and the comparison with available data. The results found and the implications for QPMSR are discussed in section 3. Finally we give our conclusions.

2 Pauli exclusion principle and the parton distributions

The behaviour at high x of $F_2^n(x)/F_2^p(x)$ [1], known since a long time, and the more recent polarization experiments [2], [3], which show that at high x the partons have spin parallel to the one of the proton, imply that $u^{\uparrow}(x)$ is the dominating parton distribution in the proton at high x.

Indeed at $Q^2 = 0$, the axial couplings of the baryon octet are fairly described in terms of the valence quarks

$$u_{val}^{\uparrow} = 1 + F \qquad u_{val}^{\downarrow} = 1 - F \quad , \tag{1}$$

$$d_{val}^{\uparrow} = \frac{1 + F - D}{2} \qquad d_{val}^{\downarrow} = \frac{1 - F + D}{2} \quad . \tag{2}$$

With the actual values [4], [5]

$$F = 0.464 \pm 0.009 \qquad D = 0.793 \pm 0.009 \quad , \tag{3}$$

$u_{val}^{\uparrow}$ is larger that the others

$$u_{val}^{\uparrow} \approx \frac{3}{2} \approx u_{val}^{\downarrow} + d_{val}^{\uparrow} + d_{val}^{\downarrow} \quad . \tag{4}$$

In a previous work we assumed that the parton distributions at a given large Q^2 depend on their abundance at $Q^2 = 0$ [6]

$$p(x) = \mathcal{F}(x, p_{val}) \quad , \tag{5}$$

with $\mathcal{F}$ an increasing function of p_{val} and with a broader shape for higher values of p_{val}. This assumption and the observation that with $F = 1/2$ and $D = 3/4$ (very near to the values quoted in Eq.(3)) one has $u^{\downarrow}_{val} = 1/2$ just on the center of the narrow range $(d^{\uparrow}_{val}, d^{\downarrow}_{val}) \equiv (3/8, 5/8)$ lead to take [7]

$$u^{\downarrow}(x) = \frac{1}{2}\left[d^{\uparrow}(x) + d^{\downarrow}(x)\right] = \frac{1}{2}d(x) \quad , \tag{6}$$

which implies

$$\Delta u(x) = u^{\uparrow}(x) - u^{\downarrow}(x) = u(x) - d(x) \quad . \tag{7}$$

Eq.(7) connects the contribution of $\Delta u(x)$ to the polarized structure function of the proton $g_1^p(x)$, with the contributions of u and d to the difference between the proton and the neutron unpolarized structure functions $F_2(x)$ [7]

$$x g_1^p(x)\Big|_{\Delta u} = \frac{2}{3}\left[F_2^p(x) - F_2^n(x)\right]_{u+d} \quad . \tag{8}$$

Since a smaller negative contribution to $g_1^p(x)$ is expected from $\Delta d(x)$ for the twofold reason that $e_d^2 = (1/4)e_u^2$ and $\Delta d_{val} \approx -(1/4)\Delta u_{val}$, Eq.(8) should hold with a good approximation for the total structure functions in the region dominated by the valence quarks: this is just the case for $x \geq 0.2$ [7].

By integrating Eq.(7) one relates Δu, which contributes to the spin sum rules, to $u-d$ contributing to the Gottfried sum rule [8]

$$I_G = \int_0^1 \frac{1}{x}\left[F_2^p(x) - F_2^n(x)\right]\, dx = \frac{1}{3}(u + \bar{u} - d - \bar{d}) = \frac{1}{3} \quad , \tag{9}$$

where the last equality follows if the sea is $SU(2)_I$ invariant ($\bar{d} = \bar{u}$). Indeed NMC experiment [9] gives for the l.h.s. of Eq.(9)

$$I_G = 0.235 \pm 0.026 \quad , \tag{10}$$

implying

$$\bar{d} - \bar{u} = 0.15 \pm 0.04 \qquad\qquad u - d = 0.85 \pm 0.04 \quad . \tag{11}$$

Many years ago Field and Feynman suggested [10] that Pauli principle disfavours the production of $u\bar{u}$ pairs in the proton with respect to $d\bar{d}$, since it contains two valence u quarks and only one d. The correlation shape-abundance for the parton distributions is also the one suggested by the Pauli principle: the most abundant parton $u^{\uparrow}$ is the one

dominating at high x and the assumption that $u^{\downarrow}$ and d have about the same shape seems confirmed by the experiment.

The role of the Pauli principle has suggested to assume Fermi-Dirac distributions in the variable x for the quark partons [6]

$$p(x) = f(x) \left[\exp\left(\frac{x - \tilde{x}(p)}{\bar{x}} \right) + 1 \right]^{-1} \quad , \tag{12}$$

where $f(x)$ is a weight function, $\bar{x}$ plays the role of the temperature and $\tilde{x}(p)$ is the *thermodynamical potential* of the parton p, identified by its flavour and spin direction. Consistently for the gluons, neglecting their polarization, one assumed [6]

$$G(x) = \frac{16}{3} f(x) \left[\exp\left(\frac{x - \tilde{x}_G}{\bar{x}} \right) - 1 \right]^{-1} \quad , \tag{13}$$

where the factor 16/3 is just the product of 2 ($S_z(G) = \pm 1$) times 8/3, the ratio of the colour degeneracies for gluons and quarks. To reduce the number of parameters, the distributions of d, s and of their antiparticles have been given in terms of the ones for u and $\bar{u}$ [6]

$$d(x) = \frac{u^{\uparrow}(x)}{1 - F} \quad , \tag{14}$$

$$\Delta d(x) = -k \; f(x) \; \exp\left(\frac{x - \tilde{x}(u^{\downarrow})}{\bar{x}} \right) \left[\exp\left(\frac{x - \tilde{x}(u^{\downarrow})}{\bar{x}} \right) + 1 \right]^{-2} \quad , \tag{15}$$

$$\bar{d}^{\uparrow}(x) = \bar{d}^{\downarrow}(x) = \bar{u}^{\downarrow}(x) \quad , \tag{16}$$

$$s(x) = \bar{s}(x) = \frac{1}{4} \left[\bar{u}(x) + \bar{d}(x) \right] \quad , \tag{17}$$

$$\Delta s(x) = \Delta \bar{s}(x) = 0 \quad , \tag{18}$$

with k fixed by the condition $\Delta d = \Delta d_{val} = F - D$. For the weight function one considered the simple form $f(x) = A \; x^{\alpha}$. In terms of seven parameters, $\bar{x}$, the $\tilde{x}$ for u and $\bar{u}$, A and α, one obtains a nice description of the unpolarized and polarized structure functions for the nucleons, but it is not possible to reproduce the the fast increasing [11] at low x of $\bar{q}(x) = \bar{u}(x) + \bar{d}(x) + \bar{s}(x)$, confirmed at very high Q^2 from the behaviour of $F_2^p(x)$ measured at H_1 [12].

Indeed, we know that the form given in Eq.(12), with different values of the $\tilde{x}$ for the different parton species, is not suitable in the limit $x \to 0$ for the most divergent part expected on general grounds to be equal for the different partons, at least in the limit

of flavour symmetry. This most divergent part should not contribute to QPMSR as the ones given by Gottfried and Bjorken [13] with $I = 1$ quantum numbers exchanged. It is therefore needed, to reproduce the data and to get information on the status of QPMSR within this approach, to disentangle the most divergent part for $x \to 0$ of the parton distributions (which does not contribute to QPMSR) from the remaining part, just given by Eq.(12).

To this extent we add a *liquid* unpolarized component, giving to the light partons, u, d and their antiparticles, the same contribution $f_L(x) = A_L\ x^{\alpha_L}(1-x)^{\beta_L}$, and to s and $\bar{s}$, as in Eq.(17), $f_L(x)/2$. To be not influenced by theoretical prejudices we consider as free parameters the $\tilde{x}$ for $u^{\uparrow(\downarrow)}$, $d^{\uparrow(\downarrow)}$, $\bar{u}$ and $\bar{d}$. Finally we introduce a new parameter in $f(x)$

$$f(x) = A\ x^{\alpha}(1-x)^{\beta} \quad . \tag{19}$$

We try also to describe the structure function

$$F_3(x) = u(x) + d(x) + s(x) - \bar{u}(x) - \bar{d}(x) - \bar{s}(x) \quad , \tag{20}$$

measured with great precision in deep inelastic reactions induced by (anti)neutrinos [14]. According to Eq.(17), we expect $s(x) - \bar{s}(x) = 0$ and thus $F_2^p(x) - F_2^n(x)$ and $F_3(x)$ to depend on the differences $u(x) - \bar{d}(x)$ and $d(x) - \bar{u}(x)$. Note that we cannot impose the conditions

$$u - \bar{u} \ = \ 2 \quad , \tag{21}$$

$$d - \bar{d} \ = \ 1 \quad , \tag{22}$$

since they would imply the Gross and Llewellyn-Smith sum rule [15]

$$\int_0^1 F_3(x)\ dx = 3 \quad , \tag{23}$$

which as well-known experimentally shows a defect. The l.h.s. of Eq.(23) is in fact measured to be 2.50 ± 0.018 (stat.) $\pm\, 0.078$ (syst.) [14], defect commonly explained in terms of QCD corrections [16].

In Figures 1-6 we compare our predictions for $F_2^p(x) - F_2^n(x)$, $xF_3(x)$, $xg_1^p(x)$ and $xg_1^n(x)$, which do not receive contributions from the *liquid* component, and for $F_2^n(x)/F_2^p(x)$

and $x\bar{q}(x)$ with the experiments. We restrict the $\tilde{x}$'s to be ≤ 1, since the factor $(1-x)^{\beta}$ in $f(x)$ makes the dependence of the distributions on $\tilde{x} \geq 1$ very smooth.

Our initial goal was to introduce spin-dependent $\tilde{x}$'s also for the $\bar{q}$'s and to test the relationship

$$\Delta\bar{u}(x) = \bar{u}(x) - \bar{d}(x) \quad , \tag{24}$$

assumed in previous works ([7] and [17]); Eqs.(7) and (24) would imply the equality

$$x g_1^p(x)\Big|_{\Delta u + \Delta\bar{u}} = \frac{2}{3}\left[F_2^p(x) - F_2^n(x)\right] \quad . \tag{25}$$

Unfortunately, we found practically the same χ^2 with negative and positive values for $\Delta\bar{u}(x)$ and/or $\Delta\bar{d}(x)$, and realized that, with $f(x)$ to be found from the data, we are unable to disentangle the contributions of $\Delta q(x)$ and $\Delta\bar{q}(x)$ to the polarized structure functions. Thus, our choice $\Delta\bar{u}(x) = \Delta\bar{d}(x) = 0$ is neither motivated by data, nor by theoretical prejudices, but simply from our present inability to get information even on their signs and to settle the important issue, relevant also for the validity of the Bjorken sum rule, whether Eq.(24) is satisfied. Eq.(25), with only Δu, Δd, $\Delta\bar{u}$ and $\Delta\bar{d}$ contributing to the polarized structure functions of the nucleons, would imply

$$x\left[g_1^p(x) - \frac{1}{4}g_1^n(x)\right] = \frac{5}{8}\left[F_2^p(x) - F_2^n(x)\right] \quad . \tag{26}$$

In Table 1 we report the parameters found here by means of the MINUIT fitting code, as well as the ones of the previous fit (without liquid) and of a one by Bourrely and Soffer [17] found on similar principles, but with several different assumptions from ours.

Indeed, the data on unpolarized nucleons structure functions are at $Q^2 = 4\ GeV^2$ [9], the neutrino data at $Q^2 = 3\ GeV^2$ [14], and $\bar{q}$ measures are performed at $Q^2 = 3\ GeV^2$ and $5\ GeV^2$ [11] and differ at small x, while our curve is intermediate between the two sets of data. The data on $g_1^n(x)$ are at $Q^2 = 2\ GeV^2$ [18], whereas $g_1^p(x)$ is measured at $Q^2 = 10\ GeV^2$ by SMC [2] and at $Q^2 = 3\ GeV^2$ by E143 [3]; despite some narrowing of the distribution at higher Q^2 showing up in the data, the values of I_p are in good agreement. This fact and the expected Q^2 dependance [17], smaller than the actual errors on the polarized structure functions, gives us confidence that our analysis is slightly affected by our neglecting the Q^2 dependance of the distributions.

The parton distributions so found are described in Figure 7. The total momentum carried by q and $\bar{q}$ is 53%. In order that gluons carry out the remaining part of the momentum $\tilde{x}_G$ is fixed to be $-1/15$. The gluon distribution is compared with the information found on them in CDHSW [19], SLAC+BCDMS [20] and in NMC [21] experiments at $Q^2 = 20\ GeV^2$ in Figure 8. The agreement is fair for $x > .1$, while the fast increase at small x, confirmed also from the data at very small x at Hera [22], confirms that a liquid component is needed also for gluons. The excess at high x of our curve with respect to experiment may be, at least in part, explained by the expected narrowing of the distribution from $Q^2 = 4\ GeV^2$, where we fit the unpolarized distributions, to $Q^2 = 20\ GeV^{2*}$.

3 Discussion of the results

The inclusion of the *liquid* term and the extension of our fit to the precise experimental results on neutrinos has brought to substantial changes in the parameters with respect to the previous work [6].

The low x behaviour of $f(x)$ become smoother ($\simeq x^{-.203\pm.013}$ instead of $x^{-0.85}$), but this is easily understood since the previous behaviour was a compromise between the smooth *gas* component and the rapidly changing *liquid* one to reproduce the behaviour of $\bar{q}(x)$. The *liquid* component, relevant only at small x, carries only .6% of parton momentum and its behaviour $\sim x^{-1.19}$, similar to the result found in [25], is less singular than the one, suggested in the framework of the multipherial approach to deep-inelastic scattering, proportional to $\sim x^{-1.5}$ [26]. The parameter $\tilde{x}(u^{\uparrow})$ took the highest value allowed by us (1.), since the factor in $f(x)$, $(1-x)^{2.34}$, is taking care to decrease $u^{\uparrow}(x)$ at high x. The temperature $\bar{x}$ is larger than the previous one and the one found by Bourrely and Soffer [17]. Instead $\tilde{x}(u^{\downarrow})$ is slightly smaller than the previous determination [6] and about half the value found in [17], where $f(x)$ is different for $u^{\uparrow}$ and $u^{\downarrow}$.

The ratio $r = u^{\downarrow}(x)/d(x)$ varies in the narrow range $(.546, .564)$ in fair agreement with the constant value $1 - F = .536 \pm .009$ assumed in [6] and slightly larger than the value

*Indeed the gluon distributions are obtained from the Q^2 dependance of the distributions according to the LAP equations [23]. In this respect it is worth noticing that the parameter Λ_{QCD} found from the corrections to the scaling is slightly smaller than the one found from different sources [4]. This qualitatively supports the idea that the evolution equations may be modified as a consequence of quantum statistical effects [24], which would favour harder quarks and softer gluons, giving rise to a slower softening of quark distributions with increasing Q^2

1/2 taken in [7] and [17].

The central value found for the first moment of $\bar{u}_{gas}(x)$, .03, is smaller than $\bar{d}_{gas}(x)/2$, .08, while Eq.(24) implies $\bar{u}(x) \geq \bar{d}(x)/2$. However, the large upper error on $\bar{u}_{gas}$ and the uncertainty in disentangling the gas and liquid contributions for the $\bar{q}$'s do not allow to reach a definite conclusion about the validity of Eq.(24).

Indeed our distributions are very well consistent with Eq.(26), as it is shown in Figure 9, where our predictions for the two sides of Eq.(26) are compared.

We have been suggested by Prof. Jacques Soffer to compare the parton distributions found here with the measured asymmetry for Drell-Yan production of muons at $y = 0$ in pp and pn reactions

$$A_{DY} = \frac{d\sigma_{pp}/dy - d\sigma_{pn}/dy}{d\sigma_{pp}/dy + d\sigma_{pn}/dy} \quad , \tag{27}$$

which at rapidity $y = 0$ reads

$$A_{DY} = \frac{(\lambda_s(x) - 1)(4\lambda(x) - 1) + (\lambda(x) - 1)(4\lambda_s(x) - 1)}{(\lambda_s(x) + 1)(4\lambda(x) + 1) + (\lambda(x) + 1)(4\lambda_s(x) + 1)} \quad , \tag{28}$$

where $\lambda_s(x) = \bar{u}(x)/\bar{d}(x)$ and $\lambda(x) = u(x)/d(x)$. At $x = .18$ we have $\lambda_s(.18) = .454$ and $\lambda(.18) = 1.748$ giving rise to $A_{DY}(.18) = -.138$ in fair agreement with the experimental result $-.09 \pm .02 \pm .025$ [27].

The behaviour of $A_{DY}(x)$ is plotted in Figure 10 together with the experimental point measured by NA51 collaboration.

We consider now the implications for the QPMSR and we begin with the one by Gross and Llewellyn-Smith [15]

$$\int_0^1 F_3(x)\, dx = u + d - \bar{u} - \bar{d} = 3 \quad . \tag{29}$$

From Table 1 we get for the l.h.s. of Eq.(29) $2.44\ ^{+.05}_{-.08}$ in good agreement with the experimental value 2.50 ± 0.018 (stat.) $\pm\, 0.078$ (syst.).

For the l.h.s. of Gottfried sum rule [8] we get

$$\frac{1}{3}(u + \bar{u} - d - \bar{d}) = .20 \pm .02 \quad , \tag{30}$$

to be compared with $.235 \pm .026$ [9]. As long as for the spin sum rules we get

$$\begin{aligned} \Delta u &= .62 \pm .02 \quad , \\ \Delta d &= -.29 \pm .04 \quad , \end{aligned} \tag{31}$$

to be compared with

$$\begin{aligned} \Delta u_{val} &= 2F = .93 \pm .02 \quad , \\ \Delta d_{val} &= F - D = -.33 \pm .02 \quad , \end{aligned} \tag{32}$$

From Eq.(31) we get

$$I_p = \frac{2}{9}\Delta u + \frac{1}{18}\Delta d = .122 \pm .007 \quad , \tag{33}$$

$$I_n = \frac{1}{18}\Delta u + \frac{2}{9}\Delta d = -.030 \pm .010 \quad , \tag{34}$$

consistent with the SMC result $.136 \pm .011 \pm .011$ [2] and the E143 result $.129 \pm .004 \pm .009$ [3] for I_p, and with the E142 result $-.022 \pm .011$ [18] for I_n.

For the Bjorken sum rule [13] we get

$$I_p - I_n = \frac{1}{6}(\Delta u - \Delta d) = .152 \pm .010 \quad , \tag{35}$$

smaller than $(1/6)|g_A/g_V| = .209$.

In Table 2. we compare our evaluations of l.h.s. QPMSR with the experiment and the prediction of theory without the QCD corrections.

By comparing the value found for the first momenta (often called by us more prosastically abundances) of the gas components of the different parton species with the r.h.s.'s of Eqs.(1) and (2), one finds

$$\begin{aligned} u^{\uparrow}_{gas} &= 1.15 \pm .01 \quad < u^{\uparrow}_{val} = 1 + F = 1.464 \pm .009 \quad , \\ d^{\downarrow}_{gas} &= .62 \pm .01 \quad \leq d^{\downarrow}_{val} = \frac{1 + D - F}{2} = .665 \pm .009 \quad , \\ u^{\downarrow}_{gas} &= .53 \pm .01 \quad \approx u^{\downarrow}_{val} = 1 - F = .536 \pm .009 \quad , \\ d^{\uparrow}_{gas} &= .33 \pm .03 \quad \approx d^{\uparrow}_{val} = \frac{1 + F - D}{2} = .335 \pm .009 \quad . \end{aligned} \tag{36}$$

The different behaviour of $u^{\uparrow}$ with respect to the other valence quark, for which $q_{gas} \sim q_{val}$, may be understood in the framework described here as an effect of Pauli blocking, since its levels are almost completely occupied differently from the other valence quarks with smaller *potentials*, as it is also shown by the fact that $\tilde{x}(u^{\uparrow})$ takes the highest value allowed. Thus, the interpretation of the defect in Gottfried sum rule as a consequence of Pauli principle, disfavouring the most abundant valence parton, $u^{\uparrow}$, seems supported by the inequalities (36). This interpretation would bring to the very relevant consequence of

a defect in the Bjorken sum rule. This conclusion is also supported by the good agreement with the data of the relationship

$$u^{\downarrow} = \frac{d}{2} + \frac{1}{2} - F \quad , \tag{37}$$

which implies

$$\Delta u = u - d + 2F - 1 \quad . \tag{38}$$

With the abundances found by us Eq.(37) reads

$$.53 \pm .01 = .51 \pm .03 \quad . \tag{39}$$

A word of caution is welcome for our conclusions on the violation of Bj sum rule, since we did not include the effect of QCD corrections in relating the quark parton distributions to the structure functions. Also we assumed no polarization for $\bar{q}$, being unable to get a reliable evaluation of $\Delta\bar{q}$ with the present precision for the polarized structure functions at small x. Indeed our description of $g_1^p(x)$ and $g_1^n(x)$ is good in terms of $\Delta u(x)$ and $\Delta d(x)$, but our prediction is smaller than the central values of the three lowest x values measured by SMC.

4 Conclusions

We compared with data the quark-parton distributions given by the sum of Fermi-Dirac functions and of a term not contributing to the QPMSR relevant at small x. We obtain a fair description for the unpolarized and polarized structure functions of the nucleons as well as for the $F_3(x)$ precisely measured in (anti)neutrino induced deep-inelastic reactions and for $\bar{q}$ total distribution. The conjectures of previous works on d distributions are well confirmed by the values chosen for their thermodynamical potentials. As long as the implications for QPMSR the values found for the first momenta of the various parton species give l.h.s.'s consistent with experiment. For the fundamental issue of the Bjorken sum rule, as advocated in previous works [6], [7] and [28], we get

$$\Delta u \approx u - d + 2F - 1 \quad , \tag{40}$$

$$\Delta d \geq F - D \quad , \tag{41}$$

to confirm the suspicion of a violation of Bjorken sum rule related to the defect in the Gottfried sum rule.

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Table 1.

Parameters	Previous fit [6]	Fit BS [17]	Actual fit $\chi^2/N = 2.47$	
A	.58		$2.66\ ^{+.09}_{-.08}$	
α	$-.85$	$-.646$ for $u^{\uparrow}_{val}$ $-.262$ for $u^{\downarrow}_{val}$	$-.203 \pm .013$	
β			$2.34\ ^{+.05}_{-.06}$	
A_L			$.0895\ ^{+.0107}_{-.0084}$	
α_L			$-1.19 \pm .02$	
β_L			$7.66\ ^{+1.82}_{-1.59}$	
$\bar{x}$	.132	.092	$.235 \pm .009$	gas abund.
$\tilde{x}(u^{\uparrow})$	.524	.510 for $u^{\uparrow}_{val}$	$1.00 \pm .07$	$1.15 \pm .01$
$\tilde{x}(u^{\downarrow})$	.143	.231 for $u^{\downarrow}_{val}$	$.123 \pm .012$	$.53 \pm .01$
$\tilde{x}(d^{\uparrow})$			$-.068\ ^{+.021}_{-.024}$	$.33 \pm .03$
$\tilde{x}(d^{\downarrow})$			$.200\ ^{+.013}_{-.014}$	$.62 \pm .01$
$\tilde{x}(\bar{u}^{\uparrow})$	$-.216$		$-.886 \pm .266$	$.015\ ^{+.034}_{-.009}$
$\tilde{x}(\bar{u}^{\downarrow})$	$-.141$		$''$	$''$
$\tilde{x}(\bar{d}^{\uparrow}) = \tilde{x}(\bar{d}^{\downarrow})$	$''$		$-.460\ ^{+.047}_{-.064}$	$.08\ ^{+.03}_{-.02}$

Table 2.

Sum rule	Experimental data	Our fit	QPM
GLS	$2.50 \pm .018 \pm .078$ [14]	$2.44\ {}^{+.04}_{-.07}$	3
G	$.235 \pm .026$ [9]	$.20 \pm .02$	1/3
EJ I_p	$.136 \pm .011 \pm .011$ [2]		$.188 \pm .005$
EJ I_p	$.129 \pm .004 \pm .009$ [3]	$.122 \pm .007$	
EJ I_n		$-.030 \pm .010$	$-.021 \pm .005$
EJ I_n	$-.022 \pm .011$ [18]		
Bj		$.152 \pm .010$	.209

Table captions

Table 1. Comparison of the values for the parameters of our best fit with the corresponding quantities, if any, found in previous analysis [6], [17].

Table 2. Comparison of our predictions for the sum rules with the experimental values and with the quark parton model (QPM) predictions without QCD corrections.

Figure captions

Figure 1. The prediction for $F_2^p(x) - F_2^n(x)$ is plotted and compared with the experimental data [9].

Figure 2. The prediction for $F_2^n(x)/F_2^p(x)$ is plotted and compared with the experimental data [9].

Figure 3. $xg_1^p(x)$ is plotted and compared with the data [2], [3].

Figure 4. $xg_1^n(x)$ is plotted and compared with the data [18].

Figure 5. $xF_3(x)$ is plotted and the experimental values are taken from [14].

Figure 6. $x\bar{q}(x)$ versus x is shown, the experimental data correspond to [11].

Figure 7. The momentum distributions of *gas* component of q and $\bar{q}$'s, and of the total *liquid* part are here shown.

Figure 8. $xG(x)$ versus x is shown, the experimental data correspond to CDHSW [19], SLAC+BCDMS [20] and NMC [21].

Figure 9. The predicted values for $x\left[g_1^p(x) - \frac{1}{4}g_1^n(x)\right]$ (dashed line), and $\frac{5}{8}\left[F_2^p(x) - F_2^n(x)\right]$ (full line) are compared.

Figure 10. The asymmetry $A_{DY}(x)$ is here plotted, the experimental result is taken from [27].

Fig. 1

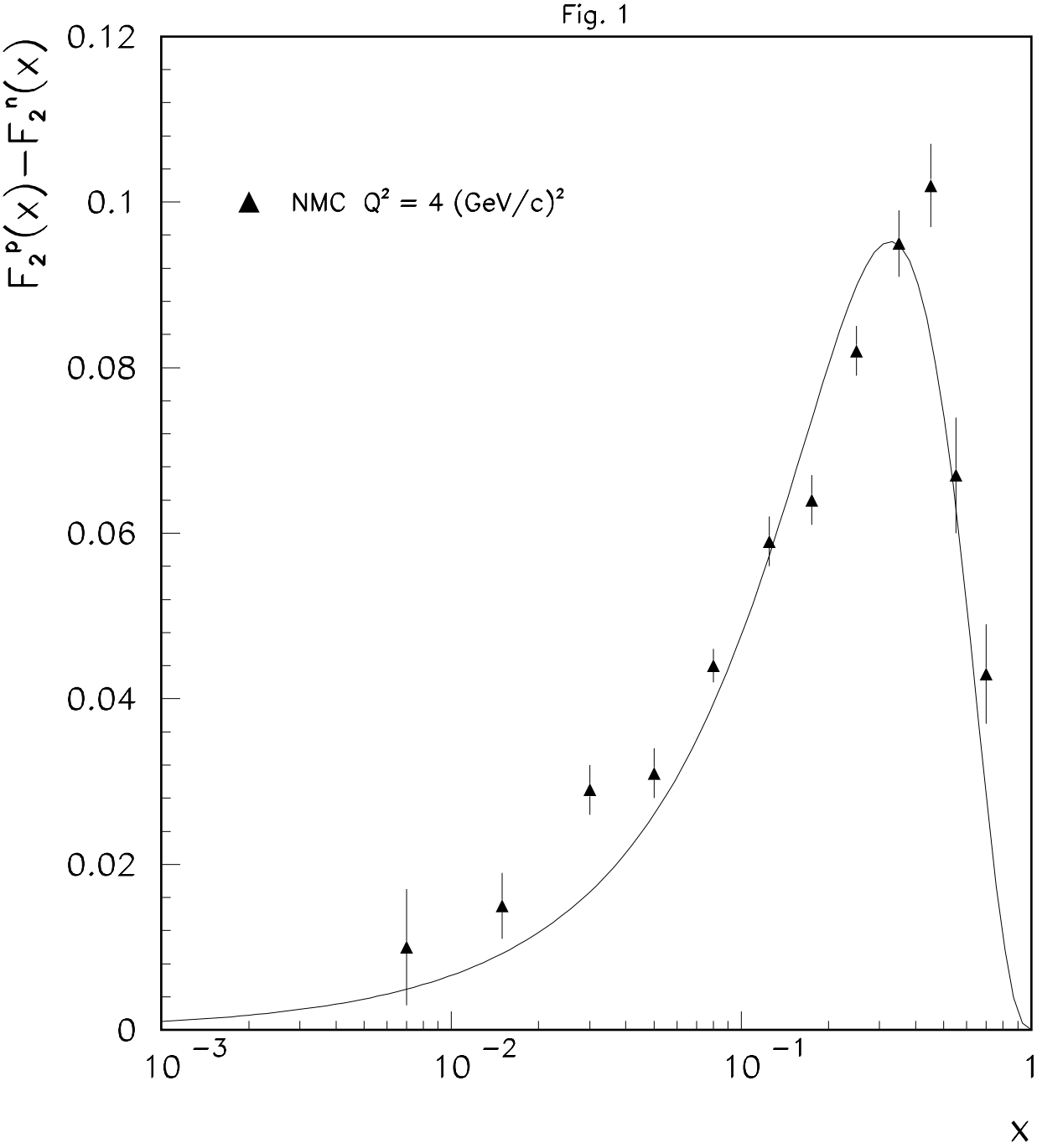


Fig. 2

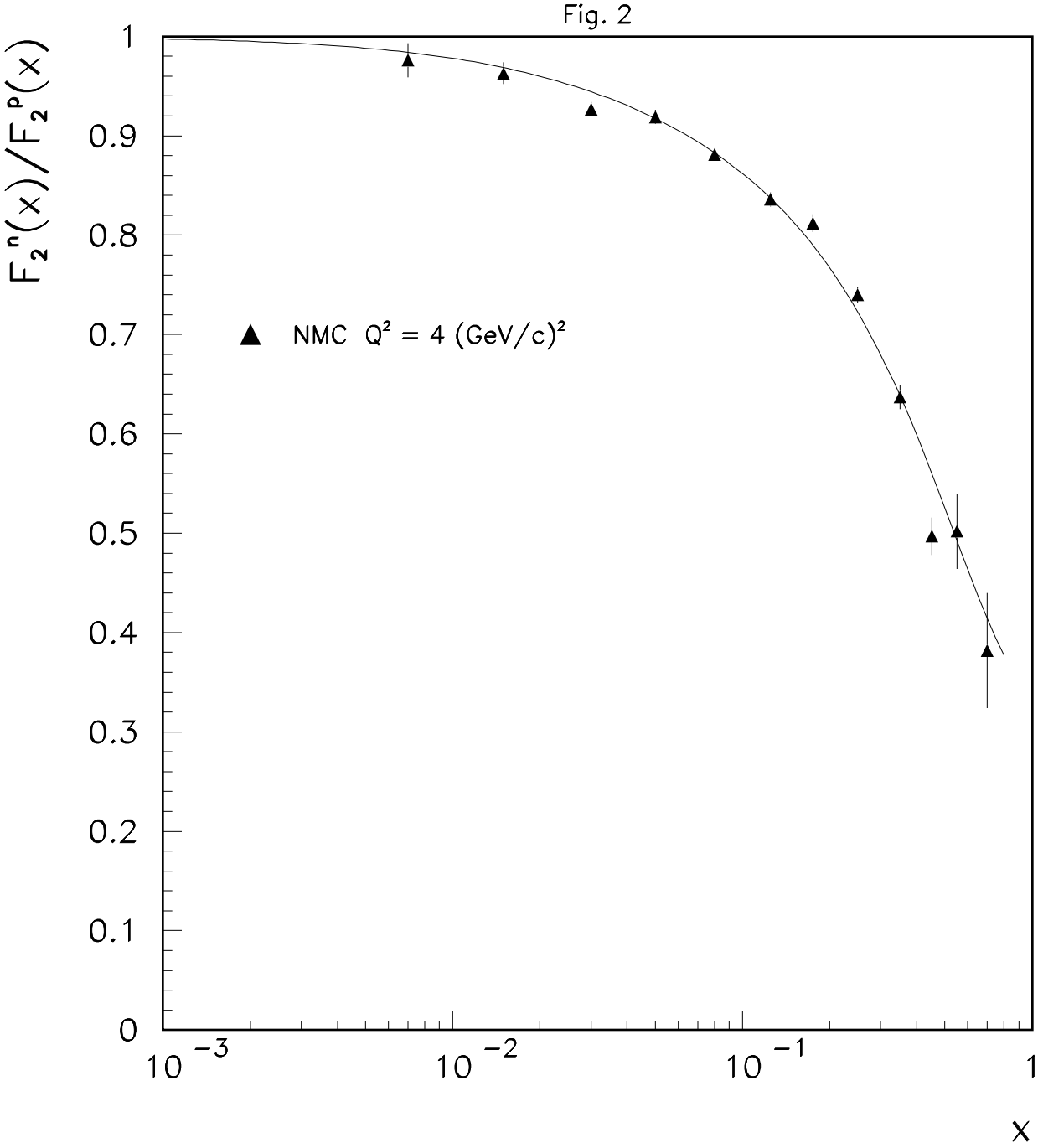

$F_2^n(x)/F_2^p(x)$
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
NMC $Q^2 = 4$ (GeV/c)^2
10^{-3}
10^{-2}
10^{-1}
1
x

Fig. 3

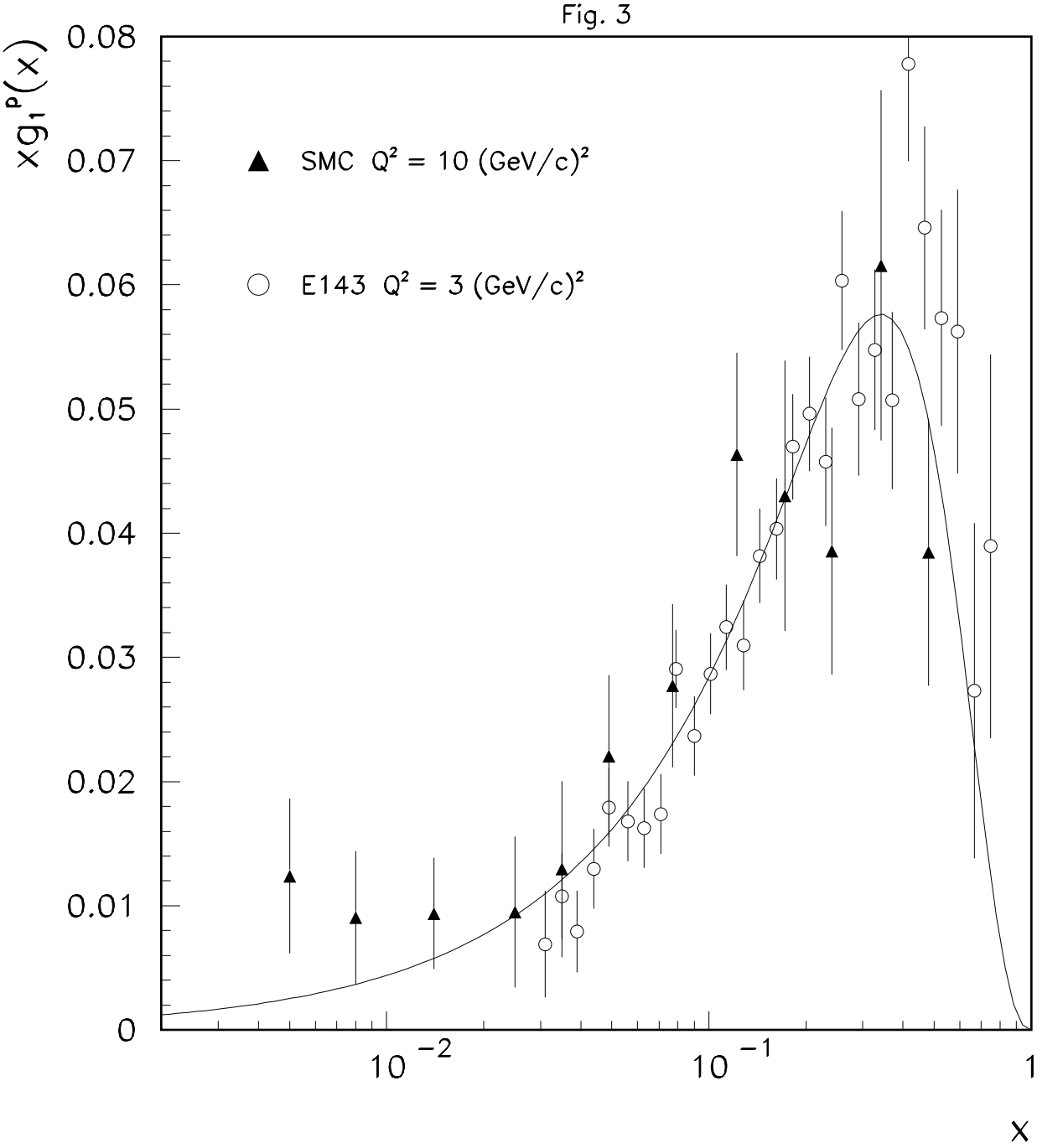

$xg_1^p(x)$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
SMC Q^2 = 10 (GeV/c)²
E143 Q^2 = 3 (GeV/c)²
10^{-2}
10^{-1}
1
x

Fig. 4

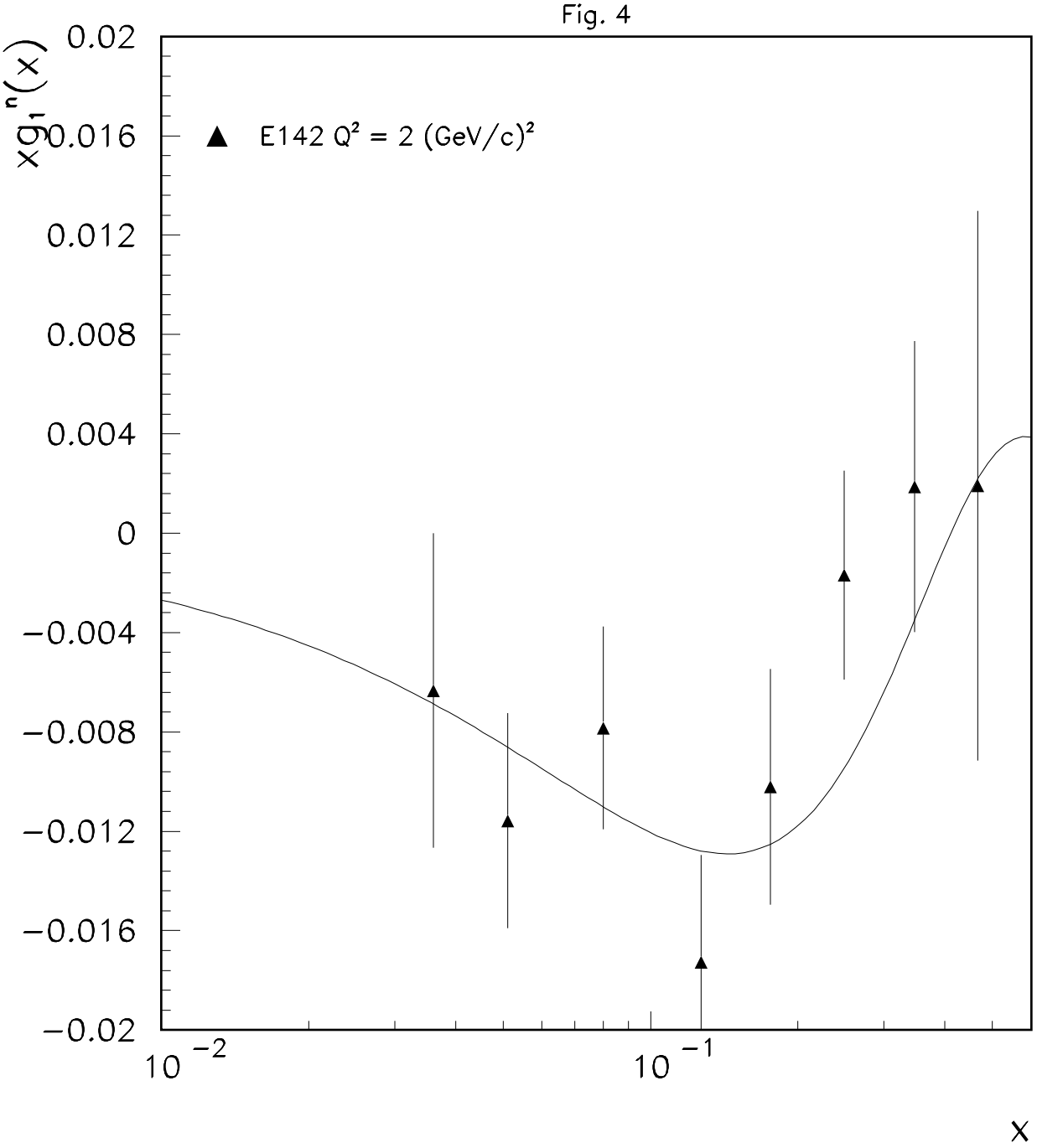

$xg_1^n(x)$
0.02
0.016
0.012
0.008
0.004
0
−0.004
−0.008
−0.012
−0.016
−0.02
E142 Q^2 = 2 (GeV/c)²
10^{-2}
10^{-1}
x

Fig. 5

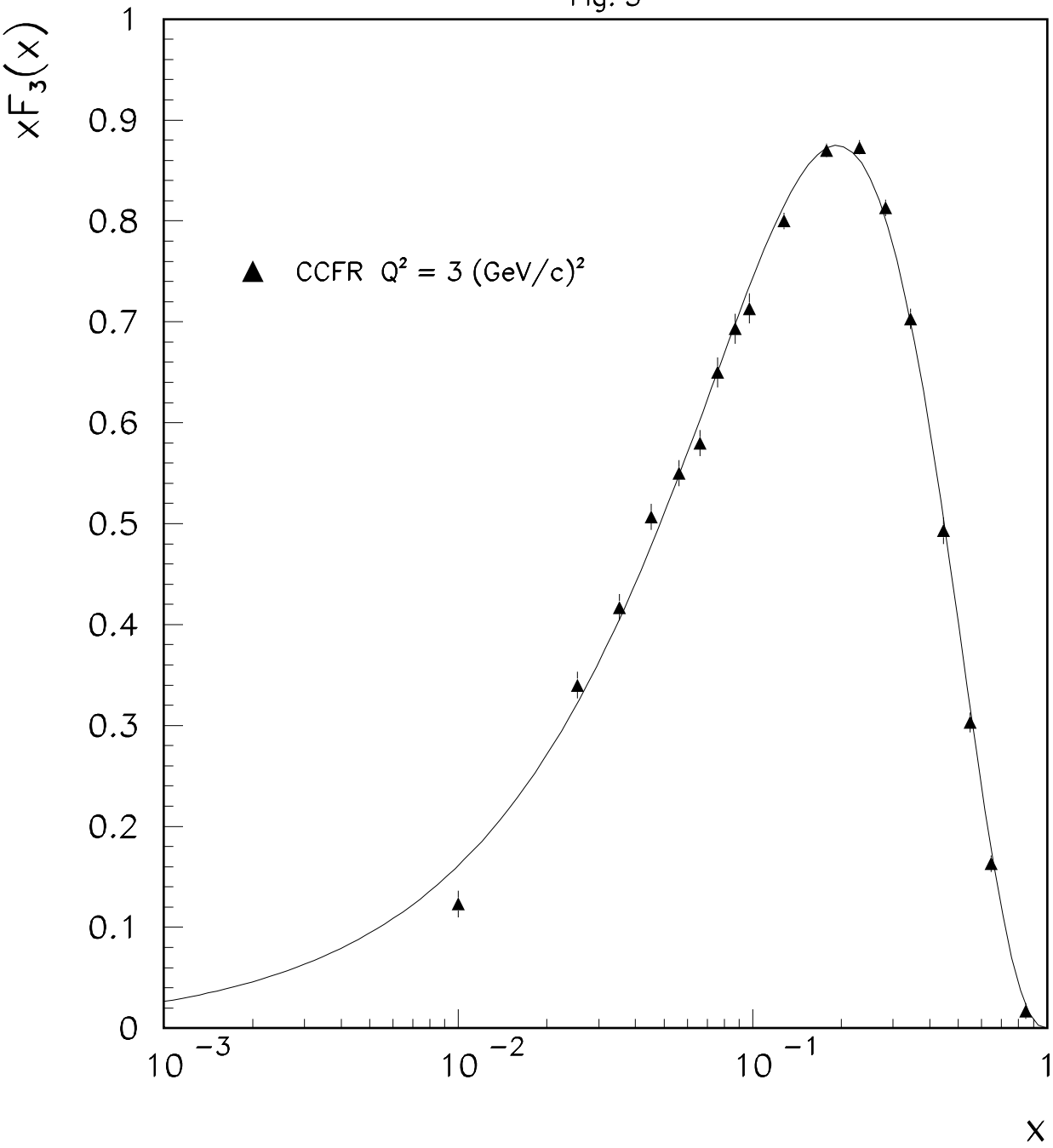


Fig.6

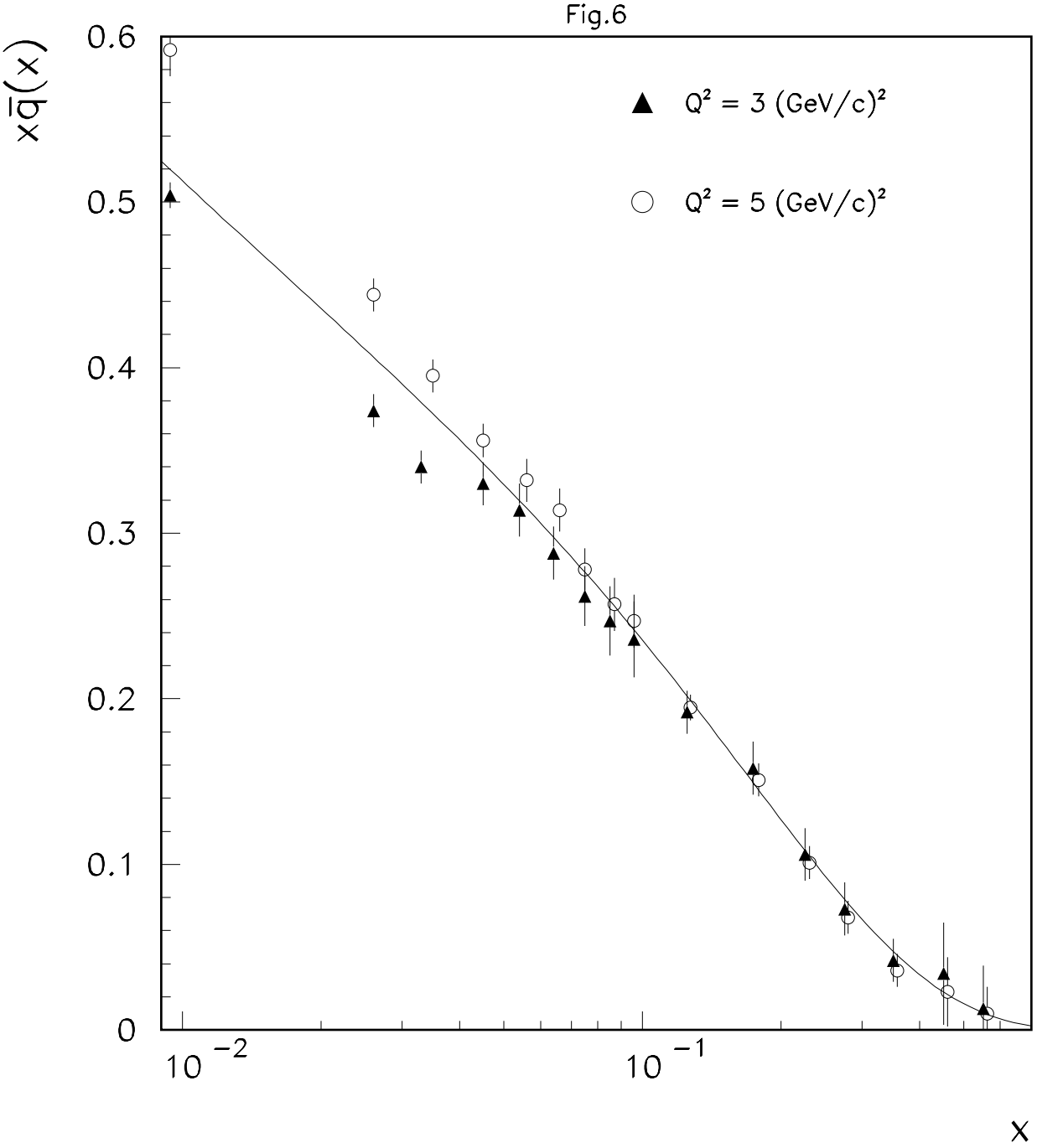


Fig. 7

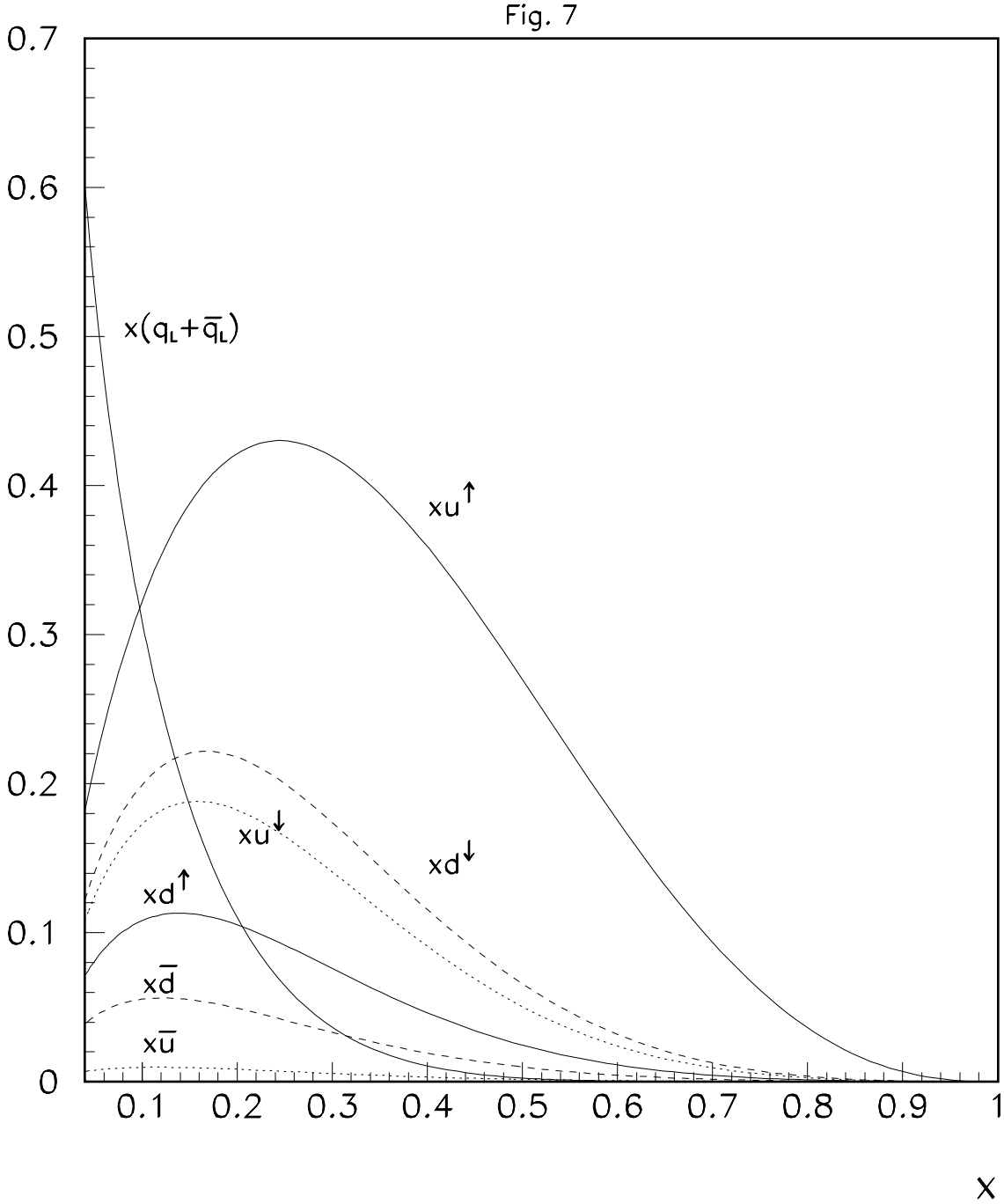

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
x
x(q_L+q̄_L)
xu↑
xu↓
xd↓
xd↑
xd̄
xū

Fig. 8

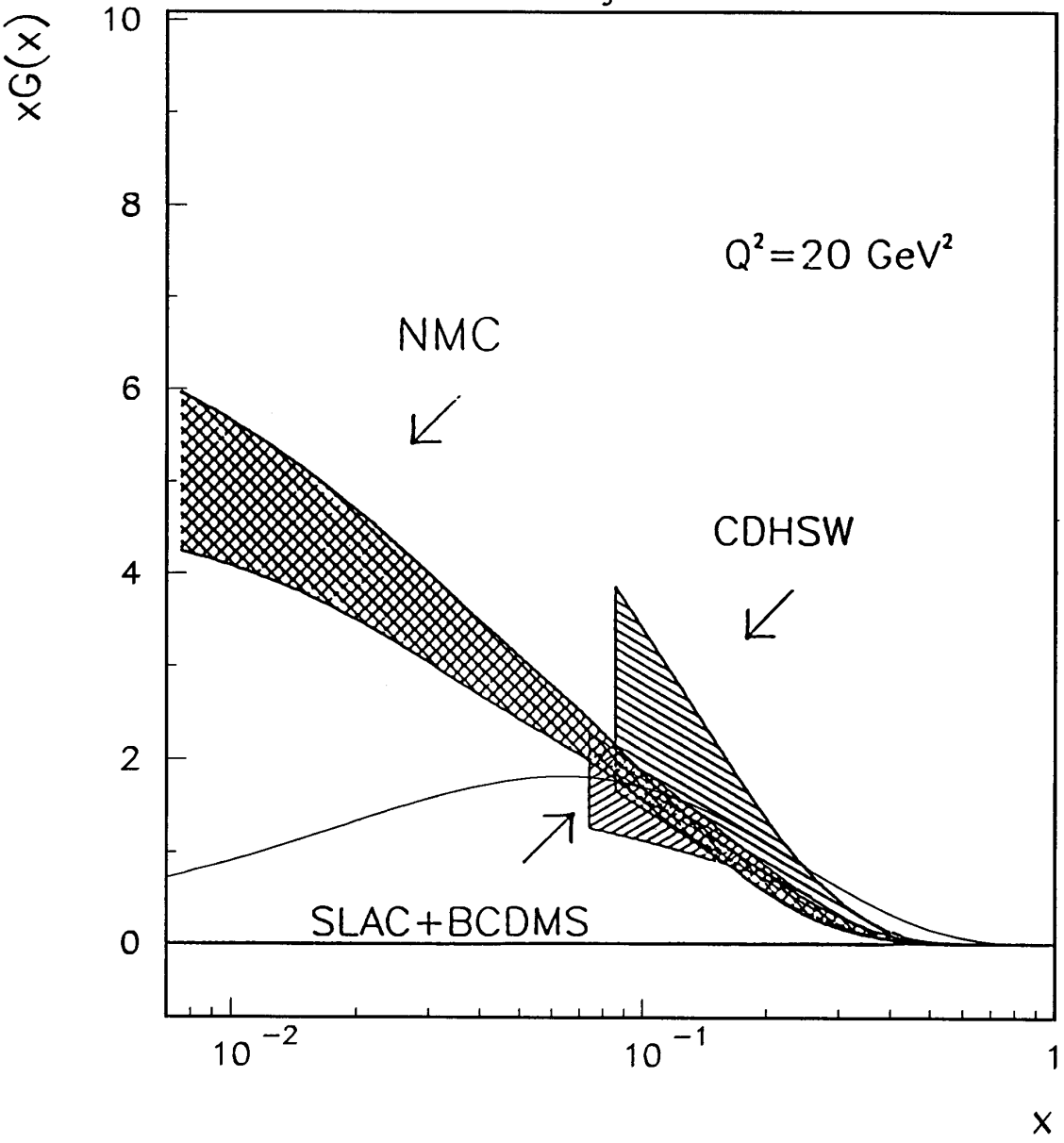


Fig. 9

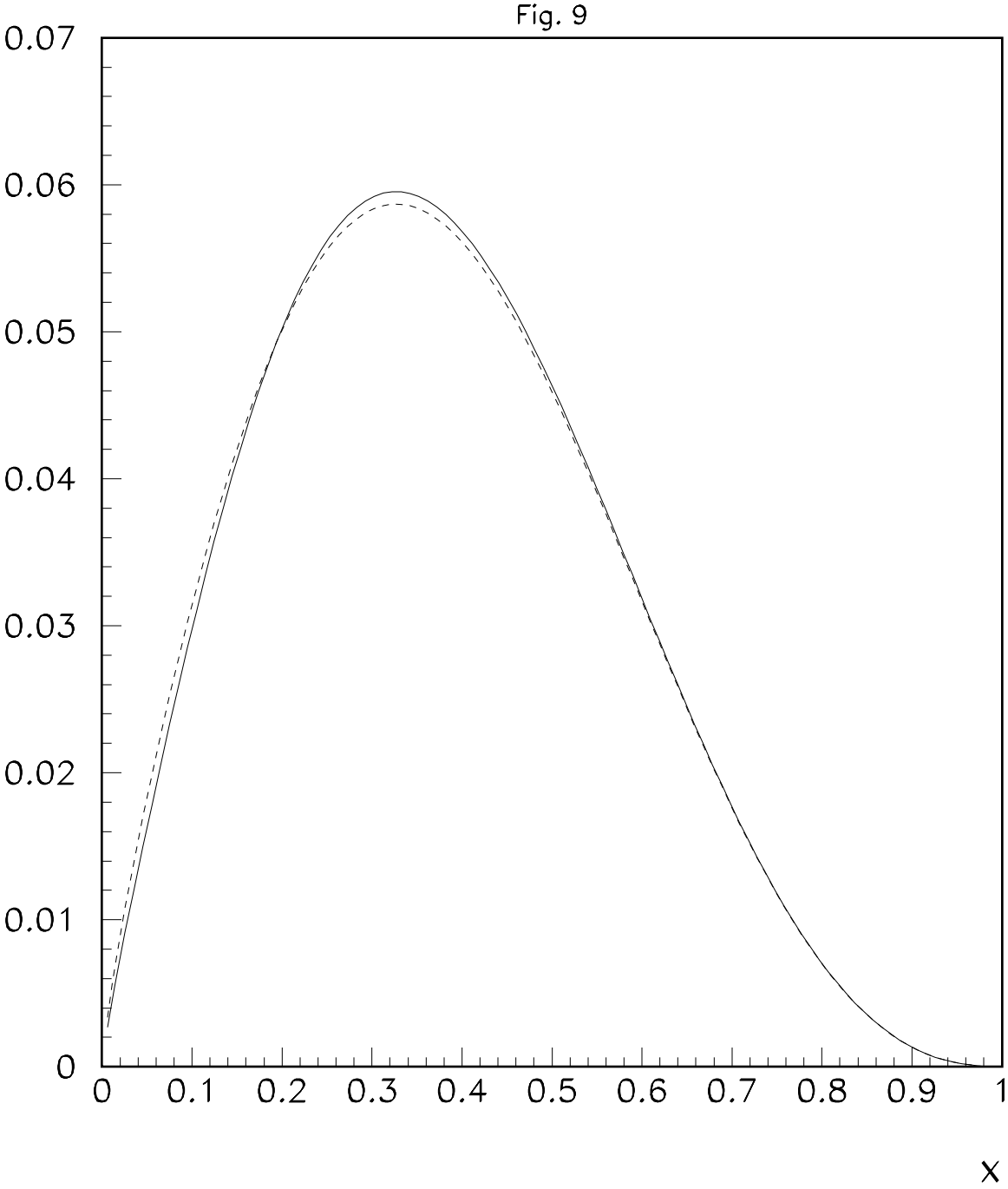
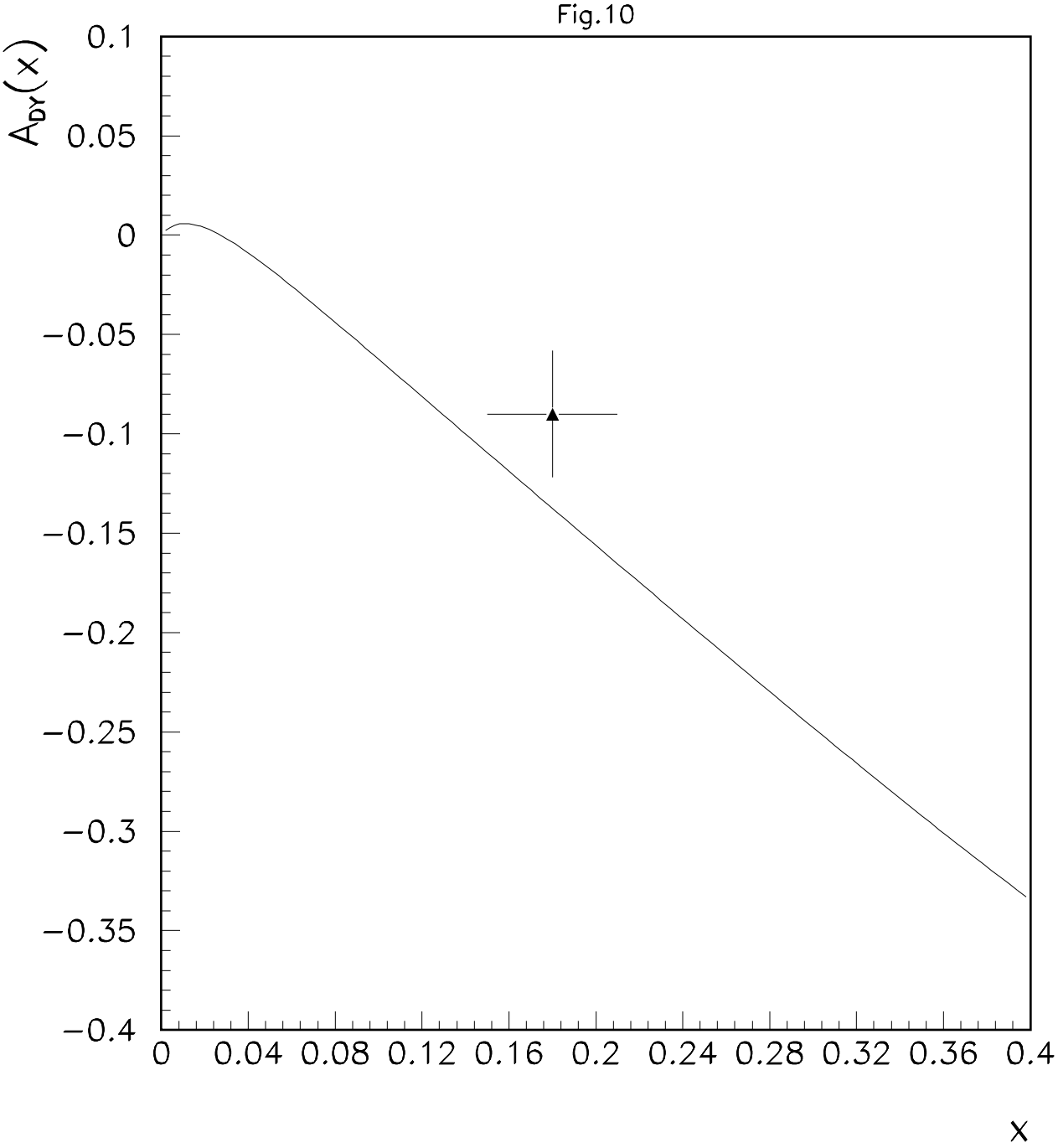


Fig.10


$A_{DY}(x)$
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−0.25
−0.3
−0.35
−0.4
0
0.04
0.08
0.12
0.16
0.2
0.24
0.28
0.32
0.36
0.4
x